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On the theory of rotation of non-axial nuclei

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Abstract. Rotation of non-axial nuclei is considered in the framework of the model suggested by Krutov in 1968. The parameters of the total deformation and non-axiality are calculated for a number of the deformed even–even nuclei with $A \geqslant 150$ on the basis of the energies of the first 2^+ rotational states. The E2 transition probability ratios are calculated for transitions between low-lying rotational levels in the non-axial nuclei. These ratios are compared with experiment (agreement is satisfactory) and with the Davydov–Chaban and Davydov–Filippov model predictions.

1. Introduction

In the papers by Krutov (1968 a, b, to be referred to as I and II respectively) a new approach to the description of the rotation of deformed nuclei was suggested, this approach being based on a definition of collective motion in the nucleus as a change of the density distribution of nuclear matter in time. On the basis of this definition the Hamiltonian of nuclear rotation was obtained with moments of inertia corresponding satisfactorily to experimental data.

In the present paper a detailed description of the rotation of the nuclei having a non-axial equilibrium shape is considered in the framework of the model suggested in I and II.

In § 2 the parameters of the total deformation and asymmetry are calculated for a number of deformed even–even nuclei with $A \geqslant 150$, using the formulae for the moments of inertia of the non-axial nucleus and the experimental values of the energies of the first rotational levels with $I^\pi K = 2^+0$ and 2^+2 (where I is the total angular momentum, K is its projection on the symmetry axis and π is the parity index). These parameters are compared with experimental data on charge distribution in the nuclei and with the predictions of the Davydov–Filippov model.

In § 3 the ratios of the reduced probabilities of the electromagnetic E2 transitions (between the low-lying rotational states with $I^\pi K = 0^+0$, 2^+0, 4^+0, 6^+0, 2^+2, 3^+2, 4^+2, 5^+2) are calculated, the results of the calculations being compared with experiment and with the predictions of the Davydov and Chaban (1960) and Davydov–Filippov (Van Patter 1960) models.

2. The moments of inertia and deformation parameters of non-axial nuclei

The coupling between the rotation and intrinsic motion is least significant in the case of the strongly deformed even–even heavy nuclei (cf. II). In the present paper we limit ourselves mainly to a consideration of these nuclei and ignore the above-mentioned coupling. The rotational Hamiltonian of the non-axial nucleus is then equal to

$$H_{\text{rot}} = \frac{\hbar^2}{2} \sum_{\nu=1}^{3} \frac{\boldsymbol{I}_\nu{}^2}{F_\nu} \tag{1}$$

where $\boldsymbol{I}_\nu$ is the angular momentum projection on the ν axis of the nucleus-fixed system (later the coordinates in this system will be designated as $\boldsymbol{r}'$ or as $x'_{\nu=1,2,3}$); F_ν is the effective moment of inertia under the rotation around the ν axis.

For the Hamiltonian (1) the energies of the first rotational levels of the nucleus with $I^\pi = 2^+$ are equal to (Davydov and Filippov 1958)

$$E_{2^+}(1,2) = \hbar \left[\sum_{\nu=1}^{3} \frac{1}{F_\nu} \mp \left\{ \left(\sum_{\nu=1}^{3} \frac{1}{F_\nu} \right)^2 + \sum_{\nu<K} \frac{1}{F_\nu F_K} \right\}^{1/2} \right]. \tag{2}$$

In accordance with the model suggested in I and II, we have

$$F_\nu = \int \tilde{\rho}_\nu\{r^2-(x_\nu')^2\}\,\mathrm{d}\boldsymbol{r}' \tag{3a}$$

$$\tilde{\rho}_\nu(\boldsymbol{r}') = \rho(\boldsymbol{r}')-\{\rho_{\min}(\boldsymbol{r}')\}_\nu \tag{3b}$$

where $\rho(\boldsymbol{r}')$ is the nuclear mass density distribution; $\{\rho_{\min}(\boldsymbol{r}')\}_\nu$ is the minimum density at the point $\boldsymbol{r}'$ under the rotation of the nucleus around the ν axis.

Assuming the uniform distribution of the density $\rho(\boldsymbol{r}')$ and using equation (3) one obtains

$$F_{1,2} = F_{\mathrm{rs}}\frac{3}{2}\left(\frac{5}{\pi}\right)^{1/2}\beta\left\{1-\frac{31}{112}\left(\frac{5}{\pi}\right)^{1/2}\beta+\frac{1\cdot 43}{\pi}\beta^2 \pm \frac{1}{\sqrt{3}}\gamma-\ldots\right\} \tag{4a}$$

$$F_3 = F_{\mathrm{rs}}\left(\frac{15}{\pi}\right)^{1/2}\beta\gamma\left\{1-\frac{4}{7}\left(\frac{5}{\pi}\right)^{1/2}\beta+\ldots\right\} \tag{4b}$$

where β is the total deformation parameter of the nucleus, γ is the non-axiality parameter and F_{rs} is the moment of inertia of a rigid sphere possessing the nuclear mass and radius ($F_{\mathrm{rs}} = \frac{2}{5}M_A R^2$). The values of the parameters β and γ of the nuclei considered, as will be seen below, ensure sufficient accuracy of the expansions performed ((4*a*) and (4*b*)).

Inserting (4*a*) and (4*b*) into (2) one obtains a system of equations for β and γ. The experimental energies $E_{2^+}(1)$ and $E_{2^+}(2)$ being known, β and γ may be obtained from these equations. They were solved numerically for a number of even–even nuclei† (the nuclear radius in F_{rs} was taken to be equal to $1{\cdot}216\,A^{1/3}$ fm). The results of the calculations of β and γ are presented in table 1.

Table 1. Parameters of the total deformation and non-axiality for the nuclear mass and charge

Nucleus	β	β_e	γ	γ_{DF}	γ_e
^{150}Nd	0·239	0·242 ± 0·010	3° 34′		12°
^{152}Sm	0·221	0·265 ± 0·007	3° 10′	13° 20′	10° 50′
^{154}Sm	0·331	0·295 ± 0·009	1° 47′		9° 30′
^{154}Gd	0·218	0·266 ± 0·009	4°	13° 50′	14° 10′
^{156}Gd	0·298	0·286 ± 0·010	2° 27′	11°	8° 40′
^{158}Gd	0·327	0·302 ± 0·010	2° 07′		9° 40′
^{160}Gd	0·338	0·310 ± 0·010	2° 20′		9° 50′
^{160}Dy	0·293	0·279 ± 0·011	2° 52′	12°	9° 40′
^{162}Dy	0·308	0·285 ± 0·008	2° 53′		11° 40′
^{164}Dy	0·349	0·296 ± 0·011	2° 53′		12°
^{164}Er	0·229	0·273 ± 0·012	4° 34′		14°
^{166}Er	0·286	0·289 ± 0·010	3° 23′	12° 40′	14°
^{168}Er	0·293	0·288 ± 0·009	3° 06′	12° 20′	13° 10′
^{170}Er	0·289	0·280 ± 0·008	2° 43′		10° 10′
^{172}Yb	0·285	0·281 ± 0·009	1° 41′	9° 20′	6° 30′
^{176}Yb	0·263	0·268 ± 0·008	2° 03′		7° 40′
^{182}W	0·202	0·220 ± 0·007	2° 38′	11° 20′	14° 10′
^{186}W	0·158	0·202 ± 0·007	5° 35′	16°	15°
^{188}Os	0·123	0·173 ± 0·009	8° 28′	19° 10′	16° 50′
^{190}Os	0·100	0·170 ± 0·008	12°	22° 20′	19°
^{192}Os	0·089	0·153 ± 0·007	14° 50′	25° 10′	20° 10′
^{194}Pt	0·055	0·140 ± 0·007	21°	30°	13° 20′
^{232}Th	0·272	0·221 ± 0·009	2° 01′		10° 40′

† In our calculation we limited ourselves to the case of the nuclei for which the relation $E_{2^+}(1)+E_{2^+}(2) = E_{3^+}(1)$ is almost satisfied. The degree of accuracy of this relation may serve (cf. II) as a criterion for ignoring the coupling between the rotation and intrinsic motion. We limited ourselves to the calculations for ^{232}Th out of all actinides, since only in this case do we possess sufficient experimental data concerning the corresponding probabilities of electromagnetic transitions (see § 3).

The values of β may be compared with those of the charge-deformation parameter β_e, derived from the quadrupole moments (Dzhelepov 1966). As is seen from table 1, the values of β and β_e are close to each other for the strongly deformed nuclei. The difference is considerable for $^{188,190,192}Os$ and ^{194}Pt, where deformations are relatively small and effects of the rotational–intrinsic motion coupling may play an important role.

Our values of γ are, as a rule, not large and are considerably smaller than the non-axiality parameters in the Davydov–Filippov model; the latter are given in the table for comparison (γ_{DF}). For the nuclei with small deformation β ($^{190,192}Os$, ^{194}Pt) the values of γ are rather large (though smaller than γ_{DF}).

3. Probabilities of the electromagnetic transitions between the rotational states of non-axial nuclei

We shall now consider the electromagnetic transitions between low rotational states of the non-axial nucleus (the scheme of the considered transitions is given in figure 1).

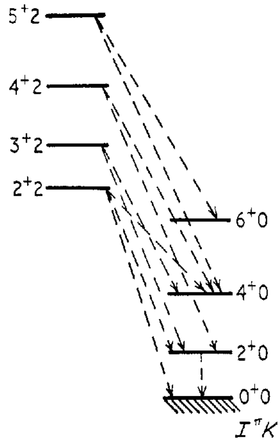

Figure 1. Scheme of the considered transitions.

The wave functions of the rotational states of the non-axial nucleus may be represented as a superposition of the symmetric top wave functions Φ^I_{MK}:

$$|IMK\rangle = \sum_{K'} A_{IKK'} \Phi^I_{MK'} \tag{5}$$

(where the conventional notations are used†), M being the angular momentum projection onto the 3rd space-fixed axis.

The reduced probabilities for E2 transitions (between two rotational levels of the non-axial nucleus) is equal to (see, for example, Davydov 1967)

$$B(\mathrm{E2}|IK \to I'K') = \frac{\pi Q_0{}^2}{2(2I+1)} \sum_{MM'\mu} |\langle I'M'K'|q_\mu|IMK\rangle|^2 \tag{6}$$

where

$$q_\mu = q_\mu{}^{(1)} + q_\mu{}^{(2)} + \ldots \tag{7a}$$

$$q_\mu{}^{(1)} = \Phi_{\mu 0}{}^2 \cos\gamma_e + (\Phi_{\mu 2}{}^2 + \Phi_{\mu -2}{}^2)\frac{\sin\gamma_e}{\sqrt{2}} \tag{7b}$$

$$q_\mu{}^{(2)} = \beta_e \frac{2}{7}\left(\frac{5}{\pi}\right)^{1/2} \left\{\Phi_{\mu 0}{}^2 \cos^2\gamma_e - (\Phi_{\mu 2}{}^2 + \Phi_{\mu -2}{}^2)\frac{\sin 2\gamma_e}{\sqrt{2}}\right\}. \tag{7c}$$

† It is worth noting that K is a good quantum number for the non-axial nuclear states (cf. II), since we have, as a rule, $|A_{IKK'\neq K}| \ll A_{IKK} \simeq 1$.

Q_0 is the intrinsic quadrupole moment, β_e- the parameter of the total deformation of charge, γ_e- the charge non-axiality parameter.

After summing over magnetic quantum numbers, we obtain the following relations for the ratios of the reduced probabilities of the transitions $IK \to I'K'$†:

$$\frac{B(\mathrm{E}2|22\to 00)}{B(\mathrm{E}2|20\to 00)} = \left(\frac{a-A_{202}b}{b-A_{202}a}\right)^2 \tag{8a}$$

$$\frac{B(\mathrm{E}2|22\to 20)}{B(\mathrm{E}2|22\to 00)} = 1{\cdot}43\left(\frac{a+2A_{202}b}{a-A_{202}b}\right)^2 \tag{8b}$$

$$\frac{B(\mathrm{E}2|22\to 20)}{B(\mathrm{E}2|20\to 00)} = 1{\cdot}43\left(\frac{a+2A_{202}b}{b-A_{202}a}\right)^2 \tag{8c}$$

$$\frac{B(\mathrm{E}2|22\to 40)}{B(\mathrm{E}2|22\to 20)} = 0{\cdot}05\left(\frac{a+9A_{202}b}{a+2A_{202}b}\right)^2 \tag{8d}$$

$$\frac{B(\mathrm{E}2|32\to 40)}{B(\mathrm{E}2|32\to 20)} = 0{\cdot}4\left(\frac{a+6A_{202}b}{a-A_{202}b}\right)^2 \tag{8e}$$

$$\frac{B(\mathrm{E}2|42\to 40)}{B(\mathrm{E}2|42\to 20)} = 2{\cdot}94\left(\frac{a+2A_{202}b}{a-5A_{202}b}\right)^2 \tag{8f}$$

$$\frac{B(\mathrm{E}2|52\to 60)}{B(\mathrm{E}2|52\to 40)} = 0{\cdot}572\left(\frac{a+8A_{202}b}{a-3A_{202}b}\right)^2 \tag{8g}$$

where

$$a = \sin\gamma_e - \beta_e\frac{2}{7}\left(\frac{5}{\pi}\right)^{1/2}\sin 2\gamma_e \tag{9a}$$

$$b = \cos\gamma_e + \beta_e\frac{2}{7}\left(\frac{5}{\pi}\right)^{1/2}\cos 2\gamma_e. \tag{9b}$$

These relations are obtained using only a linear (in coefficients $A_{IKK'}$) approximation, the coefficient A_{IKK} being put equal to unity in these relations, but give results of good accuracy for all the nuclei considered. The coefficients $A_{IKK'}$ are derived from the system of equations that is obtained by using the Hamiltonian (1) and wave functions (5). The expressions for the transitions into the states with $I^\pi K = 4^+0$; 6^+0 and from the states with $I^\pi K = 3^+2$; 5^+2 are obtained in the framework of perturbation theory (the operator $H_{\mathrm{int}}{}^{\mathrm{B}}$ from II was considered as the perturbation). It is for the sake of convenience in comparing the ratios of the reduced probabilities that we express all $A_{IKK'}$ in terms of A_{202}, which can be done when γ is small.

The charge non-axiality parameter γ_e may differ noticeably from the mass non-axiality parameter γ. Therefore (for the calculations of the ratios of the reduced probabilities of electromagnetic transitions), the parameters γ_e were obtained from equation (8*a*) using the experimental values of the ratio $B(\mathrm{E}2|00\to 22)/B(\mathrm{E}2|00\to 20)$ taken from Dzhelepov (1966) and Van Patter (1960). The values γ_e are given in table 1.

It is worth noting that the following simple relation

$$\tan^2\gamma_e \simeq \frac{B(\mathrm{E}2|00\to 22)}{B(\mathrm{E}2|00\to 20)} \tag{10}$$

is convenient for approximate estimates of γ_e.

The ratios of the reduced probabilities of E2 transitions were calculated using the equations (8*b*–*g*) and the values of γ_e, A_{202} and β_e determined by the above method. The

† We consider it necessary to point out that a factor of 2 was omitted by mistake in formula (18) in I ($5/32\pi$ should be replaced by $5/16\pi$).

results of our calculations are represented in figures 2–7 by a full line. The experimental values were taken from the following papers: Begjanov and Rackovyzky (1965), Casten *et al.* (1967), Davydov (1968), Grigoriev and Avotina (1960), Starodubzev and Rackovyzky (1966) and Van Patter (1960), and are plotted using triangles and dots. The data presented show satisfactory agreement between our theory and experiment, especially taking into account the considerable errors of the experimental results. In addition, as the ratio $B(\mathrm{E2}|00 \to 22)/B(\mathrm{E2}|00 \to 20)$ (from which the parameter γ_e is determined) is measured with considerable error, the parameters of charge non-axiality γ_e given in table 1 may change when the experimental results are defined more exactly. This fact may lead to some modification of the theoretical values of the other ratios of the reduced probabilities of electromagnetic transitions.

The results of calculations of the reduced probability ratios in the framework of the Davydov–Chaban model are taken from Davydov (1968) and are represented in figures 2, 4–7 by a broken line. The comparison with the Davydov–Filippov model is carried out

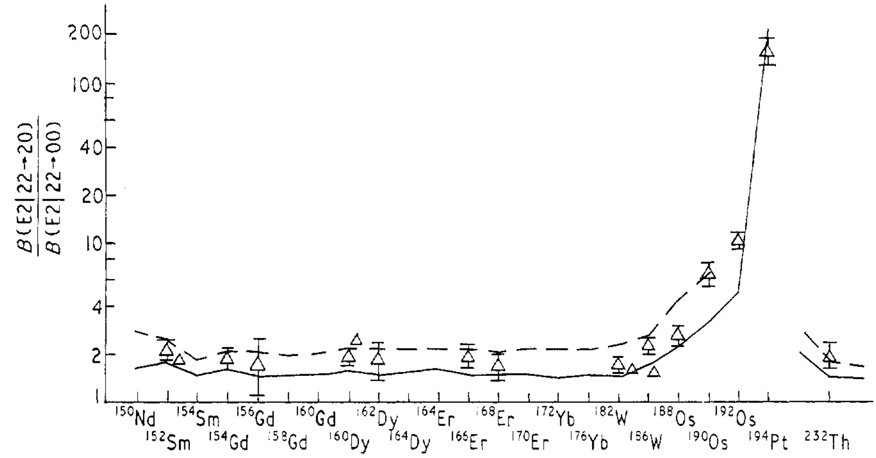

Figure 2. Ratio of the reduced probabilities for the E2 transitions $22 \to 20$ and $22 \to 00$. The results of calculations in accordance with equation (8*b*) are represented by a full line; a broken line corresponds to the results obtained in accordance with the Davydov–Chaban model. Triangles represent the experimental data.

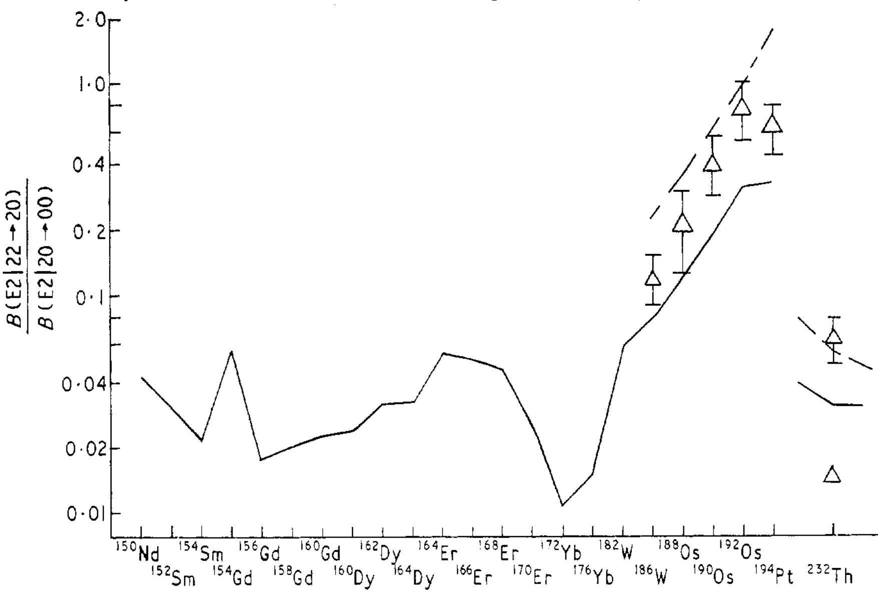

Figure 3. The same as figure 2, but for the transitions $22 \to 20$ and $20 \to 00$ (equation (8*c*)). Calculations in accordance with the Davydov-Filippov model are represented by a broken line.

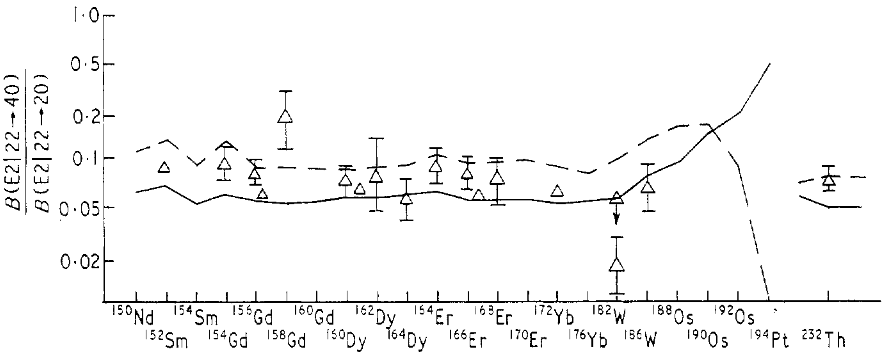

Figure 4. The same as figure 2, but for the transitions 22 → 40 and 22 → 20. A continuous line was calculated in accordance with equation (8*d*).

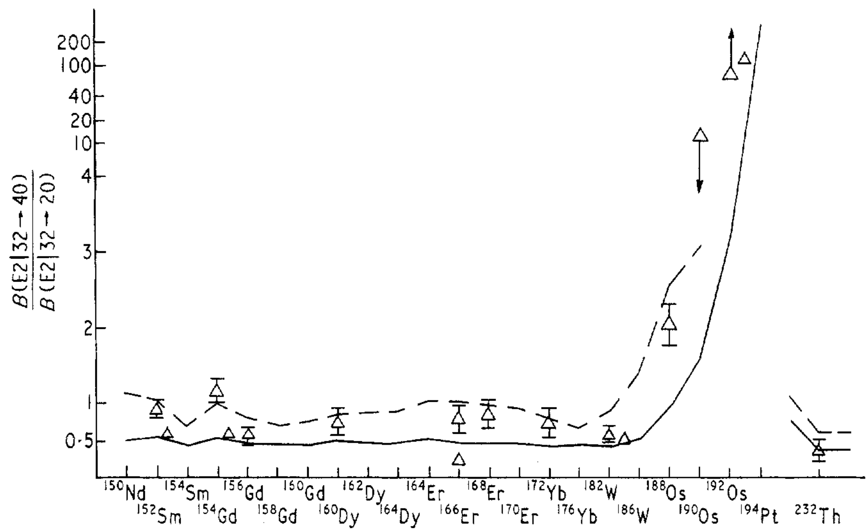

Figure 5. The same as figure 2, but for the transitions 32 → 40 and 32 → 20 (equation (8*e*)).

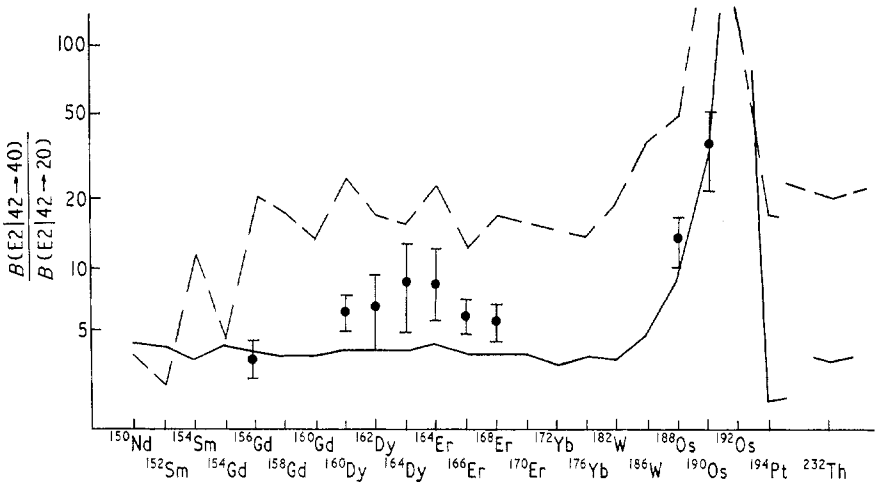

Figure 6. The same as figure 2, but for the transitions 42 → 40 and 42 → 20 (equation (8*f*)). Full circles correspond to the experimental data that were taken from Davydov (1968).

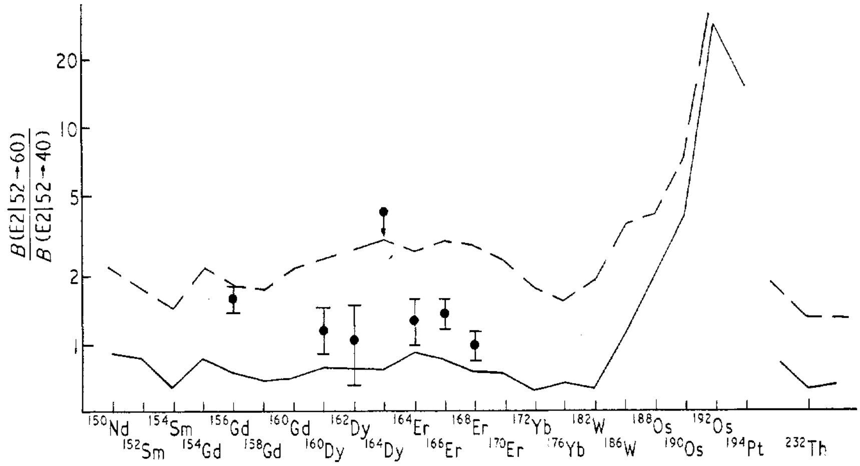

Figure 7. The same as figure 2, but for the transitions 52 → 60 and 52 → 40 (equation (8*g*)). The experimental data denoted by full circles were taken from Davydov (1968).

in figure 3 (broken line). The results of calculations in the framework of this model are taken from Van Patter (1960)†.

The Davydov–Filippov–Chaban model is regarded at present as giving the best description of the experiments considered. As is seen from the figures, the agreement of our results is no worse than using this model. Our theory has the advantage that the parameter β is given by the model (from the moments of inertia) which is not so in the Davydov–Filippov–Chaban model. However, the total number of parameters used is the same in both models. Indeed, in the Davydov–Filippov–Chaban model γ_e is considered to be equal to γ, while in our model $\gamma_e \neq \gamma$ but is found from $B(\mathrm{E2}|00 \to 22)/B(\mathrm{E2}|00 \to 20)$. As is seen from table 1, the values of γ_e and γ in our approach differ significantly, so that their difference derived from other independent experiments (independent of the positions of the 2^+ levels and E2 transitions from them) may serve as a criterion for the choice of the model. For example, the measurement of gyromagnetic factors of the rotational states and the measurement of admixtures of M1 transitions between these states may serve as such independent experiments. Their theoretical treatment is carried out by the authors in the following paper (Krutov and Zackrevsky 1969).

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† As far as we know, calculations of $B(\mathrm{E2}|22 \to 20)/B(\mathrm{E2}|20 \to 00)$ in the framework of the Davydov–Chaban model have not been carried out. It should be noted that the calculations in the framework of the Davydov–Chaban and Davydov–Filippov models were carried out considering only the $q_\mu^{(1)}$ term in the quadrupole moment operator.

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